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THE ROBUSTNESS OF THE STUDENT T TEST WHEN SAMPLING FROM A WEIBULL DISTRIBUTION

by

David P. Allen

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THESIS

THE ROBUSTNESS OF THE STUDENT t TEST WHEN SAMPLING FROM A WEIBULL DISTRIBUTION

by

David P. Allen

September 1970

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The Robustness of the Student t Test When Sampling from a Weibull Distribution

by

David P. Allen
Captain, United States Marine Corps
B.S., State University at Brockport, New York, 1964

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

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ABSTRACT

When testing with the t-test, it is assumed that the sample under investigation is from a normal population. The purpose of this thesis is to examine the sensitivity of the t-test to violations of this normality assumption. A computer simulation was performed to draw sets of 10,000 samples from an infinite Weibull population. A t-test was performed on each sample to test the null hypothesis H_o: $\mu \leq \mu_o$ where μ_o was the true mean of the Weibull population. The number of times that H_o was rejected was recorded for all combinations of eight levels of significance, samples ranging in size from 2 to 31, and for values of the parameters of the distribution $\lambda = 1,2,3$ and $\beta = 1,2,3$.

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I. INTRODUCTION

A. GENERAL

In statistical experimentation it is sometimes desired to test the assumption that the mean, μ, of a statistical population is in some way related to a hypothesized value μ_o. To evaluate the assumption, a null hypothesis H_o is formulated about μ, and tested against an alternative hypothesis, H, by taking a sample from the population under investigation and forming the t-statistic $t = \frac{\bar{x} - \mu_o}{S/\sqrt{n}}$. In this transformation $\bar{x} = \frac{1}{n} \sum_{k=1}^{n} x_k$ is the sample average, $S = \frac{1}{n-1} \sum_{k=1}^{n} (x_k - \bar{x})^2$ is the unbiased sample standard deviation, n is the sample size, and μ_o is the hypothesized population mean.

The cumulative t-distribution has been tabled for various values of n, and levels of significance α. When testing the null hypothesis H_o: $\mu \leq \mu_o$ against the alternative hypothesis H: $\mu > \mu_o$, the procedure is to reject H_o if the calculated t is greater than or equal to the tabled (critical) $t_{(n-1)(1-\alpha)}$ for n-1 degrees of freedom at the 1-α conficence level where $t_{(n-1)(1-\alpha)}$ is obtained from the upper tail of the t-distribution. Similarly, if the null hypothesis H_o: $\mu \geq \mu_o$ is tested against H: $\mu < \mu_o$, the null hypothesis is rejected if $t \leq -t_{(n-1)(1-\alpha)}$ where $-t_{(n-1)(1-\alpha)}$ is obtained from the lower tail of the t-distribution. In the case of a two trailed test, the null hypothesis H_o: $\mu = \mu_o$

is tested against H: $\mu \neq \mu_o$ and is rejected if $t \leq -t_{(n-1)(1-\alpha/2)}$ or if $t \geq t_{(n-1)(1-\alpha/2)}$ [Ref. 5].

B. BACKGROUND AND PURPOSE

When testing with the t-test it is assumed that the sample under investigation is from a normal population. In general, the purpose of this thesis is to examine the sensitivity of the t-test to certain violations of this normality assumption.

Some previous writers, e.g. Bartlett [1] have investigated the theoretical distribution of the t statistic, when sampling from an infinite non-normal population. Bartlett concludes from his study that even though his work was incomplete, and not of much quantitative value, it does indicate that for moderate departures from normality the t-test may still be used with confidence, particularly for testing differences in means of equal numbers of observations.

In a different approach, Pearson [6] describes how he mechanically drew samples from an infinite non-normal population. In the case where the means of only two samples were being tested for equivalence, the value of t was calculated for each sample to empirically obtain some idea of the frequency distribution. Using a chi square test to fit the observed t to a theoretical t distribution did not appear to bring out any systematic discrepancy. Taken as a whole the values of χ^2 were higher than should be expected if the variation from theory was solely due to chance. Also

the fits on the whole were better for larger size samples. However, Pearson never drew more than 1000 samples. A greater number of samples is needed to determine the five percent point, and more so the one percent point, since the number of rejections at these levels is so low.

Specifically, this thesis examined the effect of sampling from a non-normal population, on the number of times the null hypothesis was rejected (given that the null hypothesis was true) when testing with the t-test. The probability of such an event is commonly known as a type I error. If the observed number of rejections obtained when sampling from a non-normal distribution is near the expected number of rejections that should be obtained by sampling from a normal distribution, it will be possible to use the t-table as if the sample had come from a normal distribution. However, if the observed number of rejections (when sampling from a non-normal distribution) is significantly different from the expected number of rejections that should have been obtained by sampling from a normal distribution, it will be necessary to adjust the procedure for using the t-table to estimate a critical t value.

C. WEIBULL DISTRIBUTION

The non-normal distribution of interest is the Weibull distribution with distribution function $f(x) = \lambda\beta x^{\beta-1} e^{-\lambda x^{\beta}}$. The expected value of the random variable X is $\lambda^{-1/\beta}\Gamma(\frac{1}{\beta}+1)$ and the variance is $\lambda^{-2/\beta}\left\{\Gamma(2/\beta+1) - [\Gamma(1/\beta+1)]^2\right\}$ [Ref.3].

The Weibull distribution frequently appears in reliability theory and life testing, where the random variable X represents the time between failures. When the shape parameter $\beta=1$ the Weibull distribution reduces to the exponential distribution which has applications in queueing theory as well as reliability theory and life testing.

The Weibull distribution takes on a variety of shapes, depending on the value of the parameter β. The spread of the distribution is determined by the value of the parameter λ. One might therefore expect that the "t-statistic" obtained by sampling from a Weibull distribution will somehow depend on the values of β and λ. If this is true, the number of rejections, given that the null hypothesis is true, will also depend on the parameter values. To examine this possibility, combinations of $\lambda=1,2,3$ with $\beta = 1,2,3$ were used to develop 9 distributions from the family of Weibull distributions. Tables I and II below give the resultant means and variances for the 9 sets of parameter values.

TABLE I

Means

λ^{β}	1	2	3
1	1.00	0.886	0.893
2	0.500	0.626	0.708
3	0.333	0.511	0.618

TABLE II

Variances

λ^{β}	1	2	3
1	1.000	0.216	0.1067
2	0.250	0.108	0.0664
3	0.111	0.072	0.0511

Figure 1 shows the shape of the Weibull distribution for the parameter values above.

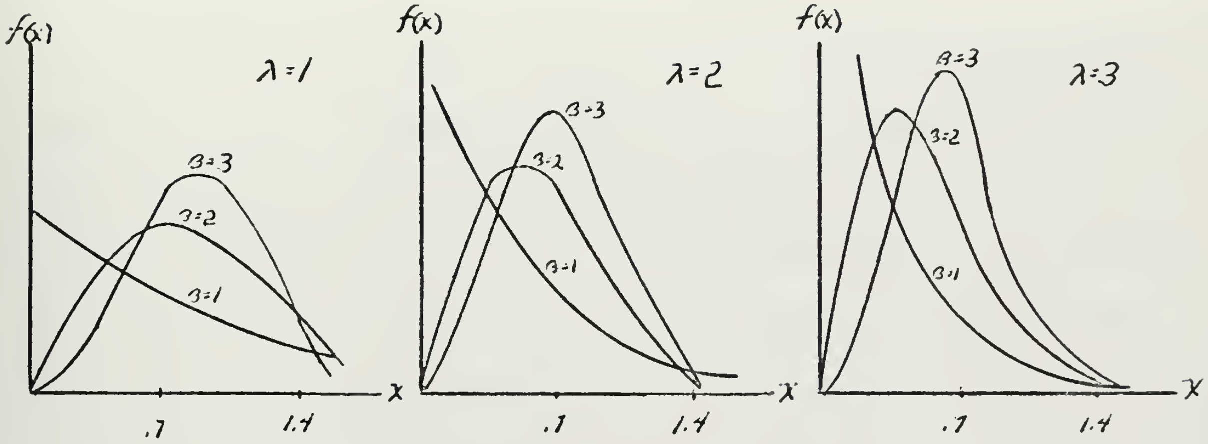

Figure 1

II. METHOD

A. GENERAL

To examine the "robustness" of the Student t-test when sampling from a Weibull distribution, a computer simulation was performed to repeatedly draw samples of size i from an infinite Weibull population. A t-statistic was calculated for each sample and used to test the null hypothesis that the sample was drawn from a population with a mean equal to or less than μ_o. To ensure that the hypothesis was in fact true, the hypothesized mean μ_o was set equal to the mean μ of the Weibull population. Each time that the null hypothesis was rejected, the result was recorded for the level of significance, α_j, at which the test was conducted, and the size of the sample. The total number of observed rejections, r_{ij}, was computed at each of eight different levels of significance α_j, $j=1,\ldots,8$ in the t-test, and for samples of size $i = 2,\ldots,31$.

For comparison, and to assist in validating the computer program, the entire experiment was repeated with sampling from a standard normal distribution. As with the Weibull case, computer simulation was used to repeatedly draw samples of size i. For each sample, a t-statistic was calculated and used to test the null hypothesis that the sample was drawn from a population with a mean μ equal to or less than μ_o. Once again the hypothesized mean μ_o was

set equal to the population mean $\mu = 0$. The number of rejections was recorded as in the Weibull case.

B. SAMPLING TECHNIQUE

Using the random number generator RANDU provided for Fortran IV with the IBM System /360 Source Library, uniform random variates were generated on the interval (0,1). J. N. Bramhall [2] in a report that discusses a comparison of three uniform random number generators for the IBM 360, fitted RANDU to a uniform (0,1) distribution with a Chi Square test at the 95% confidence level.

The uniform random variates obtained from RANDU were subsequently used to produce Weibull random variates by the inverse transformation method [4]. Since the cumulative frequency distribution F(x) ranges over the interval (0,1), the uniform (0,1) random variates, V, that were generated from RANDU were set equal to F(x). Solving the resultant equation for x produces random variates with the distribution function desired.

In the case of the Weibull distribution $F(x) = 1 - e^{-\lambda x^{\beta}}$. Setting $F(x) = V$ and solving for x yields

$$F(x) = V = 1 - e^{-\lambda x^{\beta}}$$

$$e^{-\lambda x^{\beta}} = 1 - V$$

$$\ell n\ e^{-\lambda x^{\beta}} = \ell n\ (1-V)$$

Since the distribution of V is symmetric

$$-\lambda x^{\beta} = \ln V$$

$$x^{\beta} = \frac{-\ln V}{\lambda}$$

$$x = \left(\frac{-\ln V}{\lambda}\right)^{1/\beta}$$

Here, V represents uniform random variates on the interval (0,1), and λ and β are again the parameters of the Weibull distribution.

Normal random variates were obtained using the subroutine GAUSS provided by the IBM System/360 Source Library. The Central Limit Theorem states that the probability distribution of the sum of n independent and identically distributed random variables X_i, with mean μ_i and variance σ_i^2 approaches asymptotically a normal distribution with mean μ and variance σ^2, where $\mu = \sum_{i=1}^{n} \mu_i$ and $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2$. Subroutine GAUSS calls RANDU to produce n uniform random uniform random variates V_i, on the interval (0,1). The expected value of the sum $E(\sum_{i=1}^{n} V_i) = \frac{n}{2}$, and the variance of the sum $Var(\sum_{i=1}^{n} V_i) = \frac{n}{12}$. Making the transformation $Z = \frac{Y - E(Y)}{\sqrt{V(Y)}} = \frac{\sum_{i=1}^{n} V_i - n/2}{\sqrt{n/12}}$ yields a standard normal random variate. A normal random variate X with any desired mean μ_x and variance σ_x^2 is obtained from $X = \sigma_x Z + \mu_x$. Subroutine GAUSS sets n at 12, which eliminates the radical in Z and speeds up the computation.

C. TESTING PROCEDURE

For each sample, the sample average and standard deviation were calculated. A t-test was then conducted on each sample to test the null hypothesis H_o: $\mu \leq \mu_o$ where μ_o was the true mean of the population from which the sample was drawn. The calculated value of t was then compared with the critical (tabled) t for the appropriate sample size $i = 2,\ldots,31$, and significance level α_j, $j = 1,\ldots,8$. If the calculated t was equal to or greater than the critical t the null hypothesis was rejected. The number of rejections r_{ij} was recorded for each level of significance α_j at which the null hypothesis was tested, and for samples of size $i = 2,\ldots,31$.

III. METHOD OF ANALYSIS

Once the number of rejections r_{ij} was determined (given that the null hypothesis was true) for each level of significance α_j, $j = 1,\ldots,8$, and for all sample sizes $i = 2,\ldots,31$, it was necessary to determine if the number of observed rejections were significantly different from the expected number of rejections e_j. The expected number of rejections, assuming a normal population, was obtained by taking the product of the probability of a type I error, α (i.e., the probability of rejecting the null hypothesis when in fact the null hypothesis is true), and the number of times the test was repeated with a different sample. For each size sample, the null hypothesis $\mu \leq \mu_o$ was tested for 10,000 different samples at the α_j level of significance. The values of α_j that were used, and the resultant expected number of rejection e_j is shown below in Table III.

TABLE III

j		2	3	4	5	6	7	8
α_j	.25	.20	.15	.10	.05	.025	.005	.0005
$e_j=(10{,}000)\alpha_j$	2500	2000	1500	1000	500	250	50	5

To determine if the observed number of rejections was significantly different from the expected number of rejections, a Chi Square test with one degree of freedom was

conducted to evaluate the null hypothesis H_o^*: $r_{ij} = e_j$ for each value of r_{ij}. Table IV is the contingency table for the Chi Square test.

TABLE IV

	Number of times H_o accepted	Number of times H_o rejected	Total
Expected number	$10000-e_j$	e_j	10000
Observed number	$10000-r_{ij}$	r_{ij}	10000

The Chi Square statistic was obtained by calculating $\chi^2 = \frac{(e_j - r_{ij})^2}{10000-e_j} + \frac{(r_{ij}- e_j)^2}{e}$. When the calculated χ^2 was greater than the critical χ^2 with one degree of freedom at the $1 - \alpha$ confidence level the null hypothesis H_o^*: $r_{ij}=e_j$ was rejected. An example of the method of analysis is given in Section VII.

IV. OUTPUT

The output obtained from the computer simulation was r_{ij}, the observed number of times that the null hypothesis H_o: $\mu \leq \mu_o$ was rejected, when in fact the null hypothesis was true. As previously indicated r_{ij} was obtained for $j = 1,\ldots,8$ levels of significance in the t-test, and for samples ranging in size from $i = 2,\ldots,31$. For each r_{ij}, the probability of a type I error, γ_{ij} was obtained by taking the ratio of the number of observed rejections to the number of samples M = 10,000 tested, e.g., $\gamma_{ij} = \frac{r_{ij}}{10000}$

Appendices A-J table the values of γ_{ij} for each of the ten cases examined (sampling from a standard normal distribution, and sampling from a Weibull distribution with nine sets of parameter values). The values of the index $i = 2,\ldots,31$ again represent the sample sizes used, and $j = 1,\ldots,8$ reference the levels of significance for which each sample was tested (see Table III). Probabilities that appear with an asterisk, i.e. γ_{ij}^{*} represent a situation where the number of observed rejections r_{ij} was significantly different from the expected number of rejections e_j at the .01 level of significance. Probabilities appearing with a check, i.e. $\gamma_{ij}^{\checkmark}$ represent the same situation at the .05 level of significance.

V. RESULTS

A. NORMAL CASE

In review, for the normal case samples were drawn from a standard normal distribution. A one tailed t-test was conducted on each of 10,000 different samples at eight levels of significance to test the null hypothesis $H_o\colon \mu \leq \mu_o$. The process was repeated for samples ranging in size from 2 to 31.

As anticipated the observed number of rejections r_{ij} did nearly equal the expected number of rejections e_j in all cases. In fact the null hypothesis $H_o^*\colon r_{ij} = e_j$ was accepted for all r_{ij} at the .01 level of significance and rejected for only eight of the 240 r_{ij} at the .05 level of significance. The rejections that did occur were attributed to the stochastic nature of the testing procedure.

Appendix A tables the results for the normal case.

B. WEIBULL CASE

The results of the simulation changed significantly when samples were drawn from a Weibull distribution.

In general, for a given value of the parameter β, and for fixed i and j, the observed number of rejections r_{ij} was relatively insensitive to changes in the parameter λ. However, for a given λ, as β increased, the r_{ij} increased causing a decrease in the number of times that $H_o^*\colon r_{ij}=e_j$ was rejected. In the three cases where $\beta = 1$, with the

exception of testing H_o: $\mu \leq \mu_o$ at the .0005 level, H_o^*: $r_{ij} = e_j$ was rejected for all r_{ij} at the .01 level. When testing H_o: $\mu \leq \mu_o$ at the .0005 level, H_o^*: $r_{ij} = e_j$ was rejected for most of the r_{ij} at the .05 level. Otherwise H_o^*: $r_{ij} = e_j$ was accepted. The results were essentially the same for the three cases where $\beta = 2$ with only a slight decrease in the total number of times H_o^*: $r_{ij} = e_j$ was rejected. In the three cases where $\beta = 3$, the results were closest to the results expected if sampling had been from a normal distribution. In fact, for $\lambda = 3$ the null hypothesis H_o^*: $r_{ij} = e_j$ was accepted for 206 of the 240 r_{ij}.

With few exceptions, the values of r_{ij} tended to increase as the sample size increased. This increasing trend was difficult to detect for small values of α, possibly because the increase was masked by the random fluctuations in r_{ij}, where the r_{ij} were already small.

A final result of the experiment was that the r_{ij} were usually less than the expected number of rejections. In fact, the stronger hypothesis H_o': $r_{ij} \leq e_j$ was accepted at the .95 level of confidence in all cases for all r_{ij}.

VI. CONCLUSIONS

The results of the experiment suggest that the validity of the t-test is sensitive to the assumption of normality if sampling is done from a Weibull distribution with the parameter values chosen in this paper. Accepting the null hypothesis H_o': $r_{ij} \leq e_j$ for any sample size and level of significance implies that the probability of rejecting a true hypothesis is less when sampling from a Weibull distribution than when sampling from a normal distribution. This will tend to cause the experimenter to announce too few significant results if the t-table is used as if sampling from a normal distribution. However, since the probability of making a false rejection γ_{ij}, when actually testing at the α level of significance has now been determined for the Weibull case, the problem of too few significant results can be overcome for any sample size by finding the critical t value corresponding to the desired level of significance γ_{ij}. This procedure will be demonstrated in an example in the next section.

In order to obtain a better estimate of the probability of rejecting a true hypothesis at the .0005 level of significance, more samples are needed. At this level of significance with 10,000 samples, the expected number of rejections is only five. The amount of deviation from the expected number of rejections was such that no rejections

frequently occurred in 10,000 samples. This implies that the probability of rejecting a true hypothesis is zero when testing at the .0005 level. However, based on the hypothesis that the observed number of rejections is equal to or less than the expected number of rejections, the probability of rejecting a true hypothesis when testing at the .0005 level is bounded between zero and .0005.

The fact that the r_{ij} increased as the sample size increased is supported by the Central Limit theorem. For large samples, the "pseudo t-distribution" formed by sampling from a Weibull distribution asymptotically approaches a normal distribution. Since this is also true of a "real t-distribution" where sampling is from a normal distribution, the observed number of rejections obtained by sampling from a Weibull distribution will approach the expected number of rejections for increasing sample sizes.

VII. EXAMPLE

To illustrate the method of analysis, and to demonstrate a procedure for using a t-table to estimate a critical t value when sampling from a Weibull distribution consider the following example.

From Appendix B the value $\gamma_{45} = .0100$ implies that for samples of size four, and testing at the .05 level of significance with a one tailed t-test, that the probability of a type one error is estimated to actually be .0100 when sampling is from a Weibull distribution. If $\gamma_{45} = .0100$ then $r_{45} = 100$ which implies 100 observed rejections of the null hypothesis H_o: $\mu \leq 1.0$. Filling in Table IV gives the results below.

	Number of times H_o accepted	Number of times H_o rejected	Total
Expected number	9500	500	10000
Observed number	9900	100	10000

The Chi Square statistic becomes $\chi^2 = \frac{(500-100)^2}{500} + \frac{(500-100)^2}{9500} =$ 336. Since 336 is larger than the critical Chi Square with one degree of freedom at either the .05 or .01 level of significance, the null hypothesis H_o^*: $r_{45} = 500$ is rejected. It can then be concluded that when sampling from a Weibull distribution with $\lambda = 1$, $\beta = 1$ and testing with a one tailed t-test at the .05 level of significance for samples of size

four, that the observed number of rejections in significantly different from the expected number of rejections had the sample been from a normal distribution. Consequently, the probability of a type I error, when sampling from the above Weibull distribution, is significantly different from the expected probability of a type I error when sampling from a normal distribution. In fact, since the stronger hypothesis H_o': $r_{45} \leq 500$ can be accepted, the probability of a type I error under the above conditions is less than the probability of a type I error when sampling from a normal distribution. As previously mentioned, this will cause the experimenter to announce too few significant results if a t-table is used as if sampling from a normal distribution. The experimenter is now faced with the problem of determining a critical value that corresponds to the probability of a type I error for samples from a Weibull distribution. Returning to Appendix B to test the null hypothesis H_o: $\mu \leq \mu_o$ for samples of size four at the .05 level of significance, where it is known that sampling is from Weibull distribution, it is observed that $\gamma_{45} = 0.05$ falls between the $\alpha = 0.15$ and $\alpha = 0.10$ columns. By entering a t-table at either the $\alpha = 0.15$ or $\alpha = 0.10$ level, for samples of size four, the experimenter will obtain an estimate of the critical value that corresponds to testing at the 0.05 level of significance for samples from a Weibull distribution. Choosing $\alpha = 0.15$ will result in a larger critical value and a more conservative test.

VIII. EXTENSIONS

The choice of the Weibull distribution was mostly arbitrary, even though reference was made to its application in reliability theory and life testing. The possibilities for extending this investigation to other non-normal distributions are numerous. In addition to the common distributions with known distribution functions, bimodal and truncated distributions warrent investigation. It would also be interesting to examine the robustness of the t-test when sampling from non-normal distributions when two means are being compared. Also of interest would be the case where the two samples are from different non-normal distributions. As indicated, the possibilities for extensions are numerous and limited only by the experimenters time, interest, and needs.

APPENDIX A

PROBABILITY OF A TYPE I ERROR, STANDARD NORMAL CASE

Sample size i	α=0.25	α=0.20	α=0.15	α=0.10	α=0.05	α=0.025	α=0.005	α=0.0005
2	.2448	.1978	.1511	.1020	.0510	.0256	.0048	.0009
3	.2491	.2017	.1533	.1015	.0499	.0259	.0061	.0008
4	.2500	.1981	.1479	.0992	.0488	.0241	.0048	.0009
5	.2495	.2013	.1545	.1025	.0514	.0262	.0053	.0002
6	.2483	.2009	.1513	.1014	.0494	.0256	.0052	.0005
7	.2454	.1980	.1504	.0997	.0493	.0253	.0054	.0006
8	.2459	.1927	.1441	.0955	.0500	.0237	.0051	.0003
9	.2491	.1974	.1479	.1024	.0536	.0277	.0056	.0009
10	.2497	.1994	.1496	.0987	.0487	.0244	.0047	.0009
11	.2533	.2028	.1503	.1004	.0515	.0261	.0049	.0009
12	.2480	.2016	.1484	.1006	.0492	.0245	.0057	.0001
13	.2406 ✓	.1913 ✓	.1426 ✓	.0943	.0507	.0265	.0051	.0005
14	.2567	.2019	.1514	.1024	.0538	.0242	.0047	.0005
15	.2475	.1981	.1440	.0967	.0483	.0261	.0045	.0001
16	.2467	.1964	.1460	.0964	.0500	.0262	.0054	.0004
17	.2429	.1947	.1437	.0927 ✓	.0459	.0227	.0041	.0009
18	.2496	.2018	.1505	.1035	.0510	.0267	.0055	.0003
19	.2434	.1940	.1426 ✓	.0970	.0479	.0238	.0040	.0005
20	.2499	.1983	.1506	.1003	.0490	.0243	.0057	.0005
21	.2450	.1944	.1427 ✓	.0932 ✓	.0460	.0223	.0050	.0004
22	.2502	.2013	.1502	.0999	.0517	.0247	.0049	.0004
23	.2448	.1973	.1468	.0986	.0500	.0246	.0045	.0006
24	.2428	.1944	.1442	.0982	.0481	.0247	.0054	.0007
25	.2523	.2020	.1533	.1012	.0470	.0232	.0052	.0005
26	.2474	.1979	.1468	.1004	.0516	.0256	.0059	.0005
27	.2557	.2027	.1523	.1003	.0500	.0264	.0050	.0005
28	.2542	.2047	.1534	.1053	.0495	.0246	.0049	.0003
29	.2522	.2032	.1542	.1032	.0528	.0284 ✓	.0048	.0006
30	.2515	.2007	.1502	.0976	.0477	.0245	.0046	.0007
31	.2441	.1949	.1470	.0965	.0481	.0238	.0044	.0004

APPENDIX B

PROBABILITY OF A TYPE I ERROR, WEIBULL CASE ($\lambda=1$, $\beta=1$)

Sample size i	$\alpha=0.25$	$\alpha=0.20$	$\alpha=0.15$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.25$	$\alpha=0.005$	$\alpha=0.0005$
2	.1355*	.1014*	.0701*	.0442*	.0221*	.0110*	.0019*	.0000✓
3	.1571*	.1043*	.0628*	.0349*	.0133*	.0078*	.0016*	.0000✓
4	.1690*	.1139*	.0695*	.0305*	.0100*	.0041*	.0010*	.0000✓
5	.1748*	.1187*	.0719*	.0348*	.0109*	.0040*	.0007*	.0000✓
6	.1810*	.1275*	.0782*	.0378*	.0110*	.0027*	.0002*	.0000✓
7	.1841*	.1302*	.0839*	.0393*	.0122*	.0041*	.0003*	.0000✓
8	.1964*	.1428*	.0903*	.0480*	.0139*	.0051*	.0004*	.0001✓
9	.1950*	.1405*	.0884*	.0456*	.0127*	.0031*	.0003*	.0000✓
10	.1967*	.1454*	.0920*	.0461*	.0158*	.0051*	.0003*	.0000✓
11	.1191*	.1410*	.0938*	.0479*	.0131*	.0031*	.0002*	.0000✓
12	.2039*	.1494*	.0960*	.0516*	.0154*	.0040*	.0000*	.0000✓
13	.1983*	.1451*	.0931*	.0489*	.0147*	.0045*	.0003*	.0001
14	.2066*	.1547*	.0990*	.0499*	.0171*	.0059*	.0005*	.0000✓
15	.2136*	.1587*	.1026*	.0546*	.0173*	.0057*	.0003*	.0000✓
16	.1997*	.1448*	.0975*	.0531*	.0175*	.0069*	.0008*	.0000✓
17	.2072*	.1533*	.1028*	.0539*	.0182*	.0056*	.0004*	.0000✓
18	.2052*	.1529*	.1032*	.0555*	.0185*	.0054*	.0002*	.0000✓
19	.2077*	.1541*	.1009*	.0542*	.0189*	.0056*	.0006*	.0000✓
20	.2129*	.1536*	.1027*	.0586*	.0187*	.0054*	.0001*	.0000✓
21	.2101*	.1532*	.1005*	.0524*	.0170*	.0059*	.0005*	.0000✓
22	.2108*	.1587*	.1039*	.0615*	.0209*	.0056*	.0004*	.0000✓
23	.2112*	.1590*	.1056*	.0590*	.0204*	.0070*	.0004*	.0000✓
24	.2150*	.1582*	.1054*	.0607*	.0185*	.0060*	.0007*	.0000✓
25	.2159*	.1629*	.1117*	.0656*	.0223*	.0070*	.0000*	.0000✓
26	.2214*	.1694*	.1118*	.0649*	.0215*	.0074*	.0002*	.0001
27	.2210*	.1653*	.1104*	.0645*	.0217*	.0069*	.0002*	.0000✓
28	.2221*	.1684*	.1153*	.0619*	.0219*	.0069*	.0003*	.0000✓
29	.2183*	.1600*	.1099*	.0659*	.0259*	.0088*	.0005*	.0000✓
30	.2185*	.1664*	.1139*	.0645*	.0219*	.0062*	.0005*	.0000✓
31	.2117*	.1578*	.1105*	.0623*	.0217*	.0063*	.0005*	.0001

APPENDIX C

PROBABILITY OF A TYPE I ERROR, WEIBULL CASE ($\lambda=1$, $\beta=2$)

Sample size i	$\alpha=0.25$	$\alpha=0.20$	$\alpha=0.15$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.025$	$\alpha=0.005$	$\alpha=0.0005$
2	.2053*	.1635*	.1209*	.0795*	.0380*	.0188*	.0041	.0000✓
3	.2125*	.1608*	.1176*	.0728*	.0345*	.0160*	.0042	.0000✓
4	.2193*	.1678*	.1191*	.0741*	.0319*	.0138*	.0029*	.0002
5	.2119*	.1645*	.1140*	.0679*	.0299*	.0135*	.0020*	.0003
6	.2213*	.1685*	.1198*	.0708*	.0321*	.0138*	.0018*	.0001
7	.2233*	.1730*	.1222*	.0745*	.0306*	.0134*	.0019*	.0001
8	.2365*	.1847*	.1328*	.0836*	.0376*	.0149*	.0015*	.0004
9	.2228*	.1710*	.1232*	.0751*	.0341*	.0154*	.0022*	.0000✓
10	.2263*	.1771*	.1270*	.0822*	.0339*	.0149*	.0017*	.0004
11	.2320*	.1824*	.1327*	.0826*	.0340*	.0150*	.0018*	.0000✓
12	.2289*	.1803*	.1316*	.0837*	.0361*	.0155*	.0019*	.0002
13	.2271*	.1756*	.1247*	.0741*	.0321*	.0137*	.0026*	.0001
14	.2359*	.1842*	.1317*	.0813*	.0362*	.0147*	.0025*	.0000✓
15	.2342*	.1853*	.1368*	.0856*	.0368*	.0149*	.0016*	.0001
16	.2320*	.1767*	.1258*	.0771*	.0338*	.0143*	.0021*	.0001
17	.2330*	.1798*	.1284*	.0793*	.0340*	.0143*	.0024*	.0000✓
18	.2288*	.1781*	.1303*	.0815*	.0352*	.0168*	.0019*	.0000✓
19	.2333*	.1806*	.1306*	.0826*	.0345*	.0159*	.0021*	.0000✓
20	.2355*	.1865*	.1331*	.0841*	.0383*	.0170*	.0027*	.0000✓
21	.2353*	.1866*	.1337*	.0831*	.0324*	.0136*	.0021*	.0000✓
22	.2400✓	.1875*	.1358*	.0870*	.0378*	.0179*	.0024*	.0000✓
23	.2357*	.1864*	.1370*	.0873*	.0364*	.0159*	.0023*	.0001
24	.2430	.1892*	.1362*	.0852*	.0363*	.0161*	.0023*	.0002
25	.2414✓	.1894*	.1384*	.0866*	.0375*	.0165*	.0030*	.0001
26	.2459	.1942	.1396*	.0856*	.0388*	.0159*	.0030*	.0002
27	.2393✓	.1880*	.1332*	.0874*	.0401*	.0175*	.0028*	.0000✓
28	.2409✓	.1934	.1391*	.0831*	.0362*	.0173*	.0018*	.0002
29	.2407✓	.1879*	.1373*	.0894*	.0386*	.0170*	.0018*	.0001
30	.2380*	.1848*	.1328*	.0830*	.0374*	.0172*	.0032*	.0002
31	.2302*	.1808*	.1292*	.0832*	.0371*	.0177*	.0036✓	.0001

APPENDIX D

PROBABILITY OF A TYPE I ERROR, WEIBULL CASE ($\lambda=1$, $\beta=3$)

Sample size i	$\alpha=0.25$	$\alpha=0.20$	$\alpha=0.15$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.025$	$\alpha=0.005$	$\alpha=0.0005$
2	.2385*	.1927✓	.1476	.0968	.0491	.0243	.0049	.0000✓
3	.2380*	.1913✓	.1429✓	.0957	.0475	.0252	.0050	.0003
4	.2442	.1929	.1459	.0954	.0452*	.0230	.0044	.0005
5	.2344*	.1839*	.1369*	.0896*	.0427*	.0212✓	.0031*	.0004
6	.2413*	.1890*	.1394*	.0894*	.0449✓	.0229	.0043	.0004
7	.2404*	.1933	.1411✓	.0940*	.0459	.0214✓	.0045	.0003
8	.2506	.2016	.1511	.1006	.0523	.0263	.0045	.0005
9	.2383*	.1895✓	.1389*	.0901*	.0437*	.0238	.0040	.0005
10	.2404✓	.1936	.1422✓	.0954	.0462*	.0232	.0046	.0008
11	.2469	.1971	.1484	.0985	.0457✓	.0222	.0039	.0004
12	.2436	.1924	.1474	.0977	.0471	.0230	.0042	.0003
13	.2418	.1888*	.1373*	.0874*	.0422*	.0198*	.0045	.0002
14	.2494	.1954	.1447	.0942	.0459	.0204*	.0051	.0004
15	.2478	.1970	.1503	.1009	.0487	.0217✓	.0033✓	.0001
16	.2433	.1905✓	.1370*	.0890*	.0434*	.0201*	.0036✓	.0003
17	.2444	.1913✓	.1408*	.0893*	.0428*	.0201*	.0047	.0004
18	.2367*	.1893*	.1406*	.0918*	.0461	.0210*	.0037	.0002
19	.2423	.1917✓	.1414✓	.0930✓	.0465	.0221	.0041	.0002
20	.2447	.1959	.1456	.0951	.0496	.0240	.0051	.0003
21	.2464	.1997	.1456	.0948	.0422*	.0201*	.0029*	.0003
22	.2482	.1982	.1461	.0969	.0485	.0235	.0055	.0004
23	.2414	.1942	.1467	.0974	.0458	.0216✓	.0038	.0002
24	.2502	.2018	.1468	.0945	.0450✓	.0226	.0036✓	.0004
25	.2511	.1975	.1458	.0949	.0455✓	.0218✓	.0051	.0002
26	.2536	.2031	.1502	.0983	.0475	.0217✓	.0047	.0004
27	.2482	.1976	.1430✓	.0972	.0477	.0233	.0050	.0001
28	.2471	.2016	.1474	.0926✓	.0450✓	.0211✓	.0039	.0004
29	.2469	.1951	.1469	.0987	.0472	.0232	.0029*	.0003
30	.2448	.1925	.1418✓	.0916*	.0437*	.0218✓	.0049	.0003
31	.2398✓	.1859*	.1374*	.0906*	.0446✓	.0229	.0047	.0007

APPENDIX E

PROBABILITY OF A TYPE I ERROR, WEIBULL CASE ($\lambda=2$, $\beta=1$)

Sample size i	$\alpha=0.25$	$\alpha=0.20$	$\alpha=0.15$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.025$	$\alpha=0.005$	$\alpha=0.0005$
2	.1357*	.1016*	.0703*	.0444*	.0222*	.0111*	.0020*	.0000✓
3	.1571*	.1043*	.0628*	.0349*	.0133*	.0078*	.0016*	.0000✓
4	.1690*	.1139*	.0695*	.0305*	.0100*	.0041*	.0010*	.0000✓
5	.1748*	.1187*	.0719*	.0348*	.0109*	.0040*	.0007*	.0000✓
6	.1810*	.1275*	.0782*	.0378*	.0110*	.0027*	.0002*	.0000✓
7	.1841*	.1302*	.0839*	.0393*	.0122*	.0041*	.0003*	.0000✓
8	.1964*	.1428*	.0903*	.0480*	.0139*	.0051*	.0004*	.0001
9	.1950*	.1405*	.0884*	.0456*	.0127*	.0031*	.0003*	.0000✓
10	.1967*	.1454*	.0920*	.0461*	.0158*	.0051*	.0003*	.0000✓
11	.1991*	.1410*	.0938*	.0479*	.0131*	.0031*	.0002*	.0000✓
12	.2039*	.1494*	.0960*	.0516*	.0154*	.0040*	.0000*	.0000✓
13	.1983*	.1451*	.0931*	.0489*	.0147*	.0045*	.0003*	.0001
14	.2066*	.1547*	.0990*	.0499*	.0171*	.0059*	.0005*	.0000✓
15	.2136*	.1587*	.1026*	.0546*	.0173*	.0057*	.0003*	.0000✓
16	.1997*	.1448*	.0975*	.0531*	.0175*	.0069*	.0008*	.0000✓
17	.2072*	.1533*	.1028*	.0539*	.0182*	.0056*	.0004*	.0000✓
18	.2053*	.1529*	.1032*	.0555*	.0185*	.0054*	.0002*	.0000✓
19	.2077*	.1541*	.1009*	.0542*	.0189*	.0056*	.0006*	.0000✓
20	.2129*	.1536*	.1027*	.0586*	.0187*	.0054*	.0001*	.0000✓
21	.2101*	.1532*	.1005*	.0524*	.0170*	.0059*	.0005*	.0000✓
22	.2108*	.1587*	.1039*	.0615*	.0209*	.0056*	.0004*	.0000✓
23	.2112*	.1590*	.1056*	.0590*	.0204*	.0070*	.0004*	.0000✓
24	.2150*	.1582*	.1054*	.0607*	.0185*	.0060*	.0007*	.0000✓
25	.2159*	.1629*	.1117*	.0656*	.0223*	.0070*	.0000*	.0000✓
26	.2214*	.1694*	.1118*	.0649*	.0215*	.0074*	.0002*	.0001
27	.2210*	.1653*	.1104*	.0645*	.0217*	.0069*	.0002*	.0000✓
28	.2221*	.1684*	.1153*	.0619*	.0219*	.0069*	.0003*	.0000✓
29	.2183*	.1600*	.1099*	.0659*	.0259*	.0088*	.0005*	.0000✓
30	.2185*	.1664*	.1139*	.0645*	.0219*	.0062*	.0005*	.0000✓
31	.2117*	.1578*	.1105*	.0623*	.0217*	.0063*	.0005*	.0001

APPENDIX F

PROBABILITY OF A TYPE I ERROR, WEIBULL CASE ($\lambda=2$, $\beta=2$)

Sample Size i	$\alpha=0.25$	$\alpha=0.20$	$\alpha=0.15$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.025$	$\alpha=0.005$	$\alpha=0.0005$
2	.2065*	.1643*	.1217*	.0801*	.0385*	.0191*	.0043	.0000✓
3	.2124*	.1644*	.1195*	.0756*	.0383*	.0167*	.0035✓	.0004
4	.2201*	.1681*	.1197*	.0746*	.0320*	.0139*	.0031	.0002
5	.2205*	.1710*	.1180*	.0700*	.0296*	.0136*	.0030*	.0003
6	.2214*	.1706*	.1215*	.0751*	.0330*	.0144*	.0020*	.0003
7	.2281*	.1770*	.1231*	.0795*	.0348*	.0142*	.0021*	.0002
8	.2377*	.1823*	.1342*	.0811*	.0351*	.0142*	.0032*	.0003
9	.2263*	.1764*	.1253*	.0752*	.0290*	.0141*	.0022*	.0000✓
10	.2269*	.1765*	.1253*	.0770*	.0356*	.0149*	.0028*	.0000✓
11	.2337*	.1805*	.1282*	.0791*	.0338*	.0144*	.0017*	.0001
12	.2306*	.1822*	.1281*	.0823*	.0361*	.0150*	.0018*	.0001
13	.2285*	.1769*	.1247*	.0774*	.0314*	.0142*	.0017*	.0001
14	.2324*	.1805*	.1306*	.0826*	.0347*	.0174*	.0017*	.0000✓
15	.2451	.1921*	.1364*	.0882*	.0399*	.0176*	.0025*	.0000✓
16	.2296*	.1759*	.1272*	.0776*	.0346*	.0153*	.0027*	.0005
17	.2343*	.1808*	.1295*	.0814*	.0354*	.0148*	.0020*	.0001
18	.2318*	.1807*	.1304*	.0831*	.0363*	.0151*	.0018*	.0001
19	.2368*	.1847*	.1301*	.0811*	.0359*	.0164*	.0027*	.0003
20	.2384*	.1888*	.1379*	.0872*	.0379*	.0154*	.0025*	.0001
21	.2372*	.1830*	.1309*	.0829*	.0347*	.0135*	.0021*	.0003
22	.2392✓	.1872*	.1370*	.0875*	.0412*	.0175*	.0028*	.0000✓
23	.2374*	.1855*	.1353*	.0863*	.0388*	.0180*	.0023*	.0000✓
24	.2404✓	.1880*	.1354*	.0856*	.0358*	.0164*	.0026*	.0002
25	.2427	.1922✓	.1385*	.0878*	.0405*	.0169*	.0018*	.0001
26	.2454	.1930	.1393*	.0875*	.0403*	.0171*	.0031*	.0002
27	.2430	.1905✓	.1404*	.0905*	.0391*	.0178*	.0028*	.0000✓
28	.2450	.1917✓	.1413✓	.0894*	.0399*	.0162*	.0023*	.0002
29	.2402	.9466	.1424✓	.0878*	.0421*	.0195*	.0022*	.0005
30	.2395✓	.1844*	.1327*	.0853*	.0364*	.0174*	.0029*	.0003
31	.2346*	.1866*	.1361*	.0876*	.0361*	.0176*	.0023*	.0002

APPENDIX G

PROBABILITY OF A TYPE I ERROR, WEIBULL CASE (λ =2, β=3)

Sample size 1	α=0.25	α=0.20	α=0.15	α=0.10	α=0.05	α=0.025	α=0.005	α=0.0005
2	.2398✓	.1935	.1486	.0971	.0492	.0240	.0049	.0000✓
3	.2399✓	.1917✓	.1438	.0965	.0475	.0253	.0050	.0003
4	.2463	.1946	.1470	.0957	.0454✓	.0234	.0044	.0005
5	.2362*	.1858*	.1384*	.0903*	.0430*	.0215✓	.0031*	.0004
6	.2437	.1903✓	.1412✓	.0905*	.0456✓	.0233	.0043	.0004
7	.2428	.1950	.1433	.0952	.0463	.0218✓	.0047	.0003
8	.2526	.2036	.1526	.1019	.0529	.0270	.0046	.0005
9	.2400✓	.1911✓	.1412✓	.0913*	.0447✓	.0242	.0042	.0005
10	.2444	.1960	.1450	.0967	.0468	.0235	.0047	.0008
11	.2509	.1999	.1508	.1002	.0466	.0222	.0041	.0004
12	.2459	.1955	.1495	.0997	.0474	.0233	.0042	.0004
13	.2444	.1929	.1405*	.0898*	.0424*	.0199*	.0045	.0002
14	.2532	.1983	.1478	.0956	.0466	.0210*	.0051	.0004
15	.2509	.1998	.1531	.1023	.0496	.0220	.0034✓	.0001
16	.2481	.1933	.1402*	.0911*	.0443*	.0206*	.0036✓	.0003
17	.2482	.1960	.1445	.0920*	.0436*	.0204*	.0048	.0004
18	.2403✓	.1922✓	.1435	.0938✓	.0471	.0216✓	.0038	.0002
19	.2474	.1947	.1447	.0944	.0479	.0225	.0043	.0002
20	.2500	.2002	.1471	.0972	.0512	.0245	.0053	.0003
21	.2501	.2023	.1487	.0970	.0439*	.0212✓	.0029*	.0003
22	.2530	.2014	.1494	.0995	.0498	.0243	.0061	.0005
23	.2462	.1980	.1505	.1006	.0477	.0226	.0039	.0002
24	.2540	.2063	.1504	.0984	.0466	.0234	.0038	.0004
25	.2557	.2017	.1495	.0977	.0473	.0223	.0054	.0003
26	.2588✓	.2074	.1537	.1009	.0493	.0224	.0049	.0005
27	.2524	.2022	.1465	.1000	.0490	.0240	.0053	.0001
28	.2520	.2056	.1522	.0952	.0468	.0216✓	.0042	.0004
29	.2521	.2002	.1501	.1014	.0488	.0237	.0034✓	.0003
30	.2504	.1962	.1454	.0942	.0454✓	.0223	.0056	.0003
31	.2442	.1917✓	.1410✓	.0927✓	.0461	.0238	.0050	.0007

APPENDIX H

PROBABILITY OF A TYPE I ERROR, WEIBULL CASE ($\lambda=2$, $\beta=3$)

Sample size i	$\alpha=0.25$	$\alpha=0.20$	$\alpha=0.15$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.025$	$\alpha=0.005$	$\alpha=0.0005$
2	.1360*	.1018*	.0703*	.0445*	.0222*	.0112*	.0020*	.0000✓
3	.1561*	.1060*	.0651*	.0373*	.0140*	.0082*	.0013*	.0002
4	.1710*	.1172*	.0712*	.0328*	.0103*	.0047*	.0007*	.0001
5	.1758*	.1240*	.0752*	.0358*	.0104*	.0036*	.0005*	.0000✓
6	.1844*	.1315*	.0821*	.0403*	.0102*	.0034*	.0003*	.0001
7	.1903*	.1352*	.0846*	.0418*	.0120*	.0032*	.0003*	.0000✓
8	.2006*	.1424*	.0910*	.0463*	.0136*	.0038*	.0004*	.0000✓
9	.1944*	.1423*	.0911*	.0444*	.0116*	.0032*	.0002*	.0000✓
10	.1955*	.1431*	.0908*	.0467*	.0148*	.0040*	.0003*	.0000✓
11	.2015*	.1492*	.0961*	.0469*	.0134*	.0040*	.0002*	.0000✓
12	.2011*	.1487*	.0969*	.0515*	.0167*	.0046*	.0000*	.0000✓
13	.2000*	.1456*	.0932*	.0506*	.0145*	.0046*	.0000*	.0000✓
14	.2047*	.1499*	.0984*	.0515*	.0179*	.0048*	.0000*	.0000✓
15	.2140*	.1602*	.1084*	.0575*	.0183*	.0059*	.0003*	.0000✓
16	.2042*	.1497*	.0966*	.0510*	.0174*	.0066*	.0006*	.0000✓
17	.2105*	.1558*	.1051*	.0538*	.0180*	.0049*	.0002*	.0000✓
18	.2027*	.1517*	.1031*	.0571*	.0194*	.0053*	.0001*	.0000✓
19	.2077*	.1541*	.1034*	.0543*	.0182*	.0059*	.0004*	.0000✓
20	.2137*	.1617*	.1055*	.0564*	.0170*	.0056*	.0003*	.0000✓
21	.2127*	.1578*	.1033*	.0567*	.0177*	.0050*	.0005*	.0000✓
22	.2159*	.1615*	.1115*	.0615*	.0196*	.0070*	.0003*	.0000✓
23	.2160*	.1616*	.1082*	.0618*	.0211*	.0071*	.0004*	.0000✓
24	.2161*	.1645*	.1095*	.0615*	.0197*	.0063*	.0003*	.0000✓
25	.2167*	.1649*	.1136*	.0633*	.0250*	.0069*	.0003*	.0000✓
26	.2222*	.1698*	.1163*	.0654*	.0204*	.0076*	.0005*	.0000✓
27	.2219*	.1671*	.1149*	.0652*	.0226*	.0082*	.0002*	.0000✓
28	.2234*	.1693*	.1153*	.0676*	.0215*	.0054*	.0004*	.0000✓
29	.2181*	.1658*	.1151*	.0659*	.0235*	.0085*	.0008*	.0000✓
30	.2127*	.1633*	.1112*	.0635*	.0230*	.0069*	.0005*	.0000✓
31	.2135*	.1612*	.1118*	.0657*	.0222*	.0073*	.0007*	.0000✓

APPENDIX I

PROBABILITY OF A TYPE I ERROR, WEIBULL CASE ($\lambda=2$, $\beta=3$)

Sample size i	$\alpha=0.25$	$\alpha=0.20$	$\alpha=0.15$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.025$	$\alpha=0.005$	$\alpha=0.0005$
2	.2068*	.1646*	.1221*	.0804*	.0388*	.0194*	.0046	.0000✓
3	.2125*	.1647*	.1196*	.0758*	.0385*	.0167*	.0035✓	.0004
4	.2183*	.1636*	.1150*	.0714*	.0327*	.0145*	.0030*	.0001
5	.2139*	.1658*	.1149*	.0684*	.0303*	.0138*	.0020*	.0003
6	.2216*	.1709*	.1215*	.0751*	.0332*	.0144*	.0020*	.0003
7	.2289*	.1748*	.1249*	.0798*	.0333*	.0136*	.0021*	.0001
8	.2348*	.1824*	.1326*	.0813*	.0330*	.0132*	.0027*	.0002
9	.2198*	.1692*	.1214*	.0754*	.0347*	.0158*	.0022*	.0001
10	.2284*	.1786*	.1279*	.0834*	.0345*	.0152*	.0017*	.0004
11	.2358*	.1844*	.1354*	.0867*	.0363*	.0144*	.0022*	.0002
12	.2327*	.1826*	.1304*	.0831*	.0361*	.0168*	.0017*	.0001
13	.2302*	.1771*	.1303*	.0799*	.0342*	.0147*	.0024*	.0001
14	.2353*	.1879*	.1334*	.0821*	.0355*	.0160*	.0024*	.0004
15	.2428	.1884*	.1368*	.0849*	.0350*	.0151*	.0030*	.0000✓
16	.2308*	.1769*	.1246*	.0780*	.0341*	.0160*	.0027*	.0002
17	.2396✓	.1857*	.1326*	.0814*	.0358*	.0141*	.0026*	.0000✓
18	.2310*	.1824*	.1318*	.0820*	.0377*	.0160*	.0019*	.0000✓
19	.2354*	.1822*	.1309*	.0828*	.0363*	.0149*	.0030*	.0000✓
20	.2387*	.1886*	.1371*	.0877*	.0360*	.0161*	.0018*	.0000✓
21	.2371*	.1863*	.1321*	.0795*	.0324*	.0137*	.0020*	.0001
22	.2438	.1901✓	.1406*	.0903*	.0393*	.0164*	.0024*	.0000✓
23	.2401✓	.1884*	.1383*	.0842*	.0371*	.0185*	.0027*	.0000✓
24	.2403✓	.1881*	.1347*	.0833*	.0344*	.0157*	.0024*	.0001
25	.2426	.1922✓	.1387*	.0866*	.0387*	.0176*	.0021*	.0001
26	.2459	.1912✓	.1409✓	.0909*	.0404*	.0182*	.0028*	.0003
27	.2435	.1931	.1391*	.0900*	.0396*	.0176*	.0022*	.0000✓
28	.2451	.1938	.1395*	.0883*	.0393*	.0175*	.0023*	.0001
29	.2448	.1959	.1405*	.0903*	.0412*	.0173*	.0026*	.0002
30	.2391*	.1919✓	.1419✓	.0899*	.0406*	.0182*	.0028*	.0000✓
31	.2335	.1841	.1348*	.0840*	.0360*	.0166*	.0022*	.0000✓

APPENDIX J

PROBABILITY OF A TYPE I ERROR, WEIBULL CASE ($\lambda=3$, $\beta=3$)

Sample size i	$\alpha=0.25$	$\alpha=0.20$	$\alpha=0.15$	$\alpha=0.10$	$\alpha=0.05$	$\alpha=0.025$	$\alpha=0.005$	$\alpha=0.0005$
2	.2409✓	.1947	.1497	.0980	.0499	.0247	.0052	.0000✓
3	.2378*	.1948	.1481	.0976	.0514	.0272	.0051	.0005
4	.2434	.1902✓	.1401*	.0940✓	.0465	.0233	.0049	.0004
5	.2375*	.1873*	.1399*	.0909*	.0436*	.0218*	.0032✓	.0004
6	.2421	.1918✓	.1428✓	.0944	.0479	.0243	.0044	.0005
7	.2486	.1970	.1471	.0979	.0493	.0246	.0045	.0003
8	.2520	.2026	.1523	.1019	.0492	.0227	.0043	.0006
9	.2359*	.1871*	.1388*	.0921*	.0468	.0249	.0048	.0004
10	.2464	.1983	.1459	.0978	.0474	.0237	.0049	.0008
11	.2518	.1996	.1534	.1038	.0524	.0227	.0044	.0006
12	.2498	.1968	.1489	.0975	.0490	.0239	.0048	.0003
13	.2440	.1921✓	.1447	.0978	.0469	.0217✓	.0042	.0002
14	.2511	.2026	.1514	.0975	.0461	.0236	.0047	.0005
15	.2591✓	.2042	.1524	.1005	.0462	.0229	.0049	.0006
16	.2455	.1917✓	.1392*	.0914*	.0444✓	.0233	.0051	.0004
17	.2541	.2016	.1484	.0993	.0464	.0213✓	.0035✓	.0002
18	.2443	.1958	.1468	.0968	.0476	.0229	.0041	.0001
19	.2506	.1983	.1456	.0945	.0474	.0224	.0049	.0001
20	.2548	.2037	.1529	.1022	.0494	.0218✓	.0044	.0001
21	.2522	.1994	.1483	.0943	.0431*	.0198*	.0040	.0004
22	.2575	.2051	.1535	.1035	.0509	.0240	.0045	.0004
23	.2548	.2048	.1507	.0997	.0475	.0241	.0054	.0003
24	.2546	.2027	.1474	.0963	.0443*	.0210*	.0043	.0007
25	.2547	.2036	.1549	.0991	.0480	.0227	.0046	.0003
26	.2593✓	.2073	.1548	.1038	.0495	.0243	.0045	.0006
27	.2568	.2069	.1541	.0997	.0486	.0248	.0044	.0001
28	.2585✓	.2055	.1540	.1002	.0471	.0226	.0044	.0002
29	.2572	.2052	.1568	.1026	.0514	.0239	.0049	.0006
30	.2559	.2067	.1529	.1012	.0487	.0261	.0046	.0004
31	.2453	.1944	.1490	.0946	.0433*	.0221	.0041	.0002

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13. ABSTRACT

When testing with the t-test, it is assumed that the sample under investigation is from a normal population. The purpose of this thesis is to examine the sensitivity of the t-test to violations of this normality assumption. A computer simulation was performed to draw sets of 10,000 samples from an infinite Weibull population. A t-test was performed on each sample to test the null hypothesis H_o: $\mu \leq \mu_o$ where μ_o was the true mean of the Weibull population. The number of times that H_o was rejected was recorded for all combinations of eight levels of significance, samples ranging in size from 2 to 31, and for values of the parameters of the distribution λ = 1,2,3 and β = 1,2,3.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
ROBUSTNESS						
STUDENT t-TEST						
t-TEST						
VIOLATION OF NORMALITY ASSUMPTION						

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